Rejecting Small Gambles Under Expected Utility[*]

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Abstract

This paper contributes to a recent debate around expected utility and risk aversion. Based on introspective rejections of small gambles, expected utility has been deemed incapable of explaining plausible behavior toward risk. We use empirical evidence to show that people often accept small gambles. In addition, we show that rejecting a gamble over a given range of wealth levels imposes a lower bound on risk aversion. Using this lower bound and empirical evidence on the range of the risk aversion coefficient, we study the relationship between risk attitudes over small-stakes and large-stakes gambles. We find that rejecting small gambles is consistent with expected utility. Paradoxical behavior is only obtained when the assumption of rejecting small gambles is made in a region of the parameter space that is not empirically relevant.

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"The report of my death was an exaggeration."
Mark Twain (May 1897)

1 Introduction

The expected utility framework has been severely criticized in a recent literature that concludes that diminishing marginal utility is an utterly implausible explanation for risk aversion.[1] The basis of the criticism can be best illustrated in Rabin (2000), who studies the relationship between risk attitudes over small and large stakes gambles under expected utility. Using his results, it is possible to present striking statements of the following kind: if a decision maker is a risk-averse expected utility maximizer and if he rejects gambles involving small stakes over a large range of wealth levels, then he will also reject gambles involving large stakes, sometimes with infinite positive returns. For instance, "suppose that, from any initial wealth level, a person turns down gambles where she loses $100 or gains $110, each with 50% probability. Then she will turn down 50-50 bets of losing $1,000 or gaining *any* sum of money," or "suppose we knew a risk averse person turns down 50-50 lose $100 or gain $105 bets for any lifetime wealth level less than $350,000 ... Then we know that from an initial wealth level of $340,000 the person will turn down a 50-50 bet of losing $4,000 and gaining $635,670." (Rabin, 2000, p. 1282).

From this paradoxical, even absurd, behavior towards large-stakes gambles, Rabin (2000) and other authors conclude that expected utility is fundamentally unfit to explain decisions under uncertainty. This paper challenges this conclusion. We show that the paradoxes identified in this literature have little empirical support.[2] In particular, we show that it is the assumption of rejecting small gambles over a large range of wealth levels, and *not* expected utility, that does not typically match real-world behavior. In articulating our response, it is more useful not to argue whether expected utility is literally true (we know that it is not, since many violations of its underpinning axioms have been exhibited).

[1] See, for example, Hansson (1988), Kandel and Stambaugh (1991), Rabin (2001), Rabin and Thaler (2001) and other references therein. Samuelson (1963), Machina (1982), Segal and Spivak (1990) and Epstein (1992) also study various issues that are related to this literature.

[2] See Watt (2002), who has independently obtained results related to ours.

Rather, one should insist on the identification of a useful range of empirical applications where expected utility is a useful model to approximate, explain, and predict behavior.

To be more specific on Rabin's criticism, let p be the proposition "agent a is a risk averse expected utility maximizer," let q be "agent a turns down the modest-stakes gamble X for a given range of wealth levels," and let r stand for "agent a turns down the large-stakes gamble Y." Then, Rabin's statements can be expressed as: if p and q hold, so does r. He then convincingly argues that most individuals do not turn down Y. This amounts to saying that r does not hold. From here, Rabin (2000) jumps to the conclusion that p does not hold. But this conclusion is not warranted unless q is either tautological or at least empirically compelling.

It is surprising that the plausibility of q as an assumption is argued in the literature purely by appealing to the reader's introspection. Introspection, however, may sometimes be misleading: what people think they *would do* in a thought experiment may turn out to be quite different from what they *actually do* when confronted with a similar real-life situation. Indeed, we shall see that q is far from being empirically evident.

Our methodology is the following. Rather than relying on introspection, we undertake two empirical quests:

(a) First, using empirical evidence from familiar small gambles that individuals frequently face, we show how in practice people often accept risks of the kind that Rabin (2000) says they reject. That is, assumption q can be directly challenged on empirical grounds. On the other hand, some people reject the same small gambles. In these circumstances, the standard practice in Economics is to estimate the model of preferences that best fits the observed data. This links to our next argument.

(b) Second, we ask whether the joint hypothesis "p and q" has ever been supported empirically in natural or experimental settings. Relying on econometric evidence from a very large number of empirical studies of actual behavior of individuals under the expected utility assumption, we find a remarkable pattern. These studies consistently indicate that the coefficient of relative risk aversion that best fits the observed behavior of individuals runs in the single-digit range. In contrast with these estimates, we find that rejecting Rabin's small bets over his assumed range of wealth levels implies double-digit and tripledigit values of the same coefficient. This explains why Rabin (2000) finds that his "p and q" joint assumption yields paradoxical implications for human behavior. Lastly, we replicate Rabin's exercise by assuming, under expected utility, the rejection of small gambles when it implies single-digit values of the coefficient of relative risk aversion, i.e., over a suitably smaller interval of wealth levels. Interestingly, we find that Expected Utility implies then no paradoxical rejections of large-stakes gambles. We conclude that paradoxical behavior is only obtained when the assumption of rejecting small gambles is made in a region of the parameter space that is not empirically relevant.

2 Accepting Small Gambles

The literature has already noted that individuals often accept, rather than reject, small gambles. For instance, "people not only engage in fair games of chance, they engage freely and often eagerly in such unfair games as lotteries ... The empirical evidence for the willingness of individuals to purchase lottery tickets, or engage in similar forms of gambling, is extensive" (Friedman and Savage, 1948, pp. 280, 286).

Consider binary gambles, in which the probability of gain $g > 0$ is denoted π and the probability of loss $l > 0$ is $(1 - \pi)$. Denote such a gamble by the tuple $[\pi; (g, l)]$. In this section we will show that in real life people often accept small gambles that are as risky as (indeed, often far riskier than) those that they perhaps say they may reject in Rabin's thought experiments. Clearly, in real life we will not often find exactly gambles such as $[0.5; (105, 100)]$ that Rabin (2000) assumes individuals reject. We will find other small gambles. Therefore, we need to introduce a notion of "riskiness" of a gamble to be able to speak of "similar" gambles in this sense.

2.1 The Riskiness of a Gamble

The most uncontroversial notion of riskiness is given by the concept of second-order stochastic dominance (SOSD); see, e.g., Mas-Colell et al. (1995, Chapter 6). Using SOSD, one could then say that two gambles are "similar" if they are not ranked according to this incomplete ordering, while one is more risky than the other if the latter SOS dominates the former. It follows from one of the well-known characterizations of SOSD (Rothschild and Stiglitz (1970)) that in the case of two gambles ordered by SOSD, all risk averse expected utility maximizers would choose unanimously one lottery over the other.

In the examples we will offer below, one could use the SOSD notion and even a weaker version of this concept (one in which the requirement of equal means is not imposed) to make our point. This incomplete ordering would render most of the gambles we will consider non-comparable and, therefore, one could term them "similar." It turns out, though, that in these examples one can make the same point using several more crude and intuitive measures of riskiness that will allow to order more gambles. We propose them at present, concentrating for simplicity on one of them.

Given a binary gamble $[\pi; (g, l)]$ with positive expected value, i.e., $\pi g - (1 - \pi)l > 0$, define its *riskiness* as the following index:

$$R(\pi, g, l) = \frac{1}{a^*},$$

where $a^* > 0$ solves $\pi e^{-a^* g} + (1 - \pi)e^{a^* l} - 1 = 0$. Note that a^* is well defined since this equation has a unique positive solution.

In the absence of an axiomatic derivation of a measure of riskiness, we will enumerate some desirable intuitive properties that this proposed measure $R(\pi, g, l)$ possesses:

- (i) Strictly decreasing in the probability of gain: if $\pi_1 > \pi_2$, $R(\pi_1, g, l) < R(\pi_2, g, l)$.
- (ii) Strictly decreasing in the gain: if $g_1 > g_2$, $R(\pi, g_1, l) < R(\pi, g_2, l)$.
- (iii) Strictly increasing in the loss: if $l_1 > l_2$, $R(\pi, g, l_1) > R(\pi, g, l_2)$.
- (iv) Homogeneity of degree 1 in gain and loss: for every $\lambda > 0$, $R(\pi, \lambda g, \lambda l) = \lambda R(\pi, g, l)$.
- (v) Consider two gambles and denote their riskiness by R_1 and R_2. If $R_1 > R_2$, the set of expected utility maximizers that would reject the gamble with riskiness R_1

strictly contains the set of expected utility maximizers that would reject the gamble with riskiness R_2.[3]

Therefore, we shall say that a gamble with positive expected value is more risky than another if it has a higher value of the index R. For our purposes, this is a simple statistic that can be computed on all the gambles we will study. Its dimension is the one of the random variable, telling us for example that Rabin's $[0.5;(105,100)]$ gamble is 10 times more risky than the $[0.5;(10.50,10)]$ gamble, a property that seems intuitive for the current discussion. In addition, this index gives us a clean measure of riskiness that abstracts from wealth considerations.

The R index just defined is by no means the only measure of riskiness we could use in the examples of the next subsection which will show that many people accept gambles that are riskier than Rabin's. For example, one could also define riskiness on the basis of other one-parameter families of preferences, or appeal to related notions found in the literature, as follows:

1. A measure based on the constant relative risk aversion (CRRA) family. Since for this family the measure of riskiness will in general depend on the consumer's wealth, one can use wealth itself to measure riskiness as follows. Consider the square root utility function as a representative of the CRRA family and for any positive-mean binary gamble $[\pi;(g,l)]$ define its riskiness as the wealth of the individual that is indifferent between accepting and rejecting the gamble. That is, the solution of the equation:

$$\pi\sqrt{w+g}+(1-\pi)\sqrt{w-l}=\sqrt{w}.$$

Since for these preferences risk aversion decreases with wealth, this also provides an intuitive measure of riskiness. The monotonicity and homogeneity properties (i–iv) stated above for index R are also shared by this new measure.

[3]To see why this property holds, note that the R index is based on the family of constant absolute risk aversion (CARA) preferences: it is actually the inverse of the coefficient of absolute risk aversion of a CARA individual indifferent between taking and not taking the gamble. See Section 3.

2. A measure based on a family of preferences that satisfy the axioms of Yaari's (1987) dual theory of choice under risk. Consider the family of preferences over lotteries $[\pi, (g, l)]$ that can be represented by the function $-l + \pi^\alpha(g + l)$ for $\alpha > 0$. It can be shown that the bigger the α the larger the level of risk-aversion exhibited by these preferences. For any lottery $[\pi, (g, l)]$ we can find the α^* of the individual that is indifferent between accepting and rejecting the lottery, and define the riskiness of the lottery as $\frac{l}{\alpha^*}$. It can be checked that this measure also satisfies properties i–iv mentioned above.

   This family is of special interest because it is based on non-expected utility preferences.

3. The variance/mean ratio. This measure is pervasive in much of the finance literature, especially in models of portfolio decisions.

In order to measure the riskiness of a gamble, one could choose any of these indices, the R index or the SOSD concept. In the examples we will discuss next, similar statements can be made using any of them.

2.2 Empirical Evidence

Out of the different ways outlined to measure riskiness of a gamble, we shall concentrate on one of them, the R index. We proceed to calculate the values of R for different small gambles. For us, as for Rabin (2000) and the related literature, "small gambles" simply means that the stakes are a small fraction of consumers' wealth.

First, we report the values of R for different Rabin gambles. Consider the gamble $[0.5; (10.50, 10)]$. It has a value of R of 210. The gamble $[0.5; (105, 100)]$ has a value of R of 2,100.

Next we report the values of R for gambles taken from daily life experiences. They show that individuals may well accept gambles whose riskiness, as measured by the R index, is greater than 2,100.

1. Cicchetti and Dubin (1994) investigate the decision of whether to purchase insurance against the risk of telephone line trouble in the home for approximately 10,000 residential

customers. The average price of this insurance was about $g = \$0.45$ a month (in some areas the price was above and in others below this amount), while the average cost of repair in case of trouble was about \$55 leading to a loss $l = 55 - 0.45$. Note that we identify the gamble with not taking the insurance: hence, g represents the savings of not paying for the insurance, and l is the net loss in the event of line trouble. The probability of line trouble was about $(1 - \pi) = 0.005$, ranging from 0.00318 to 0.00742. A 42.9% of users in the sample chose not to buy insurance, so these users were taking a small gamble. The R index associated with the average consumer facing the gamble $[0.995, (0.45, 54.55)]$ is 59.64. However, it is not difficult to see that many households were actually taking a gamble whose R index was far greater than 2,100. For instance, if the cost of repair is just a few dollars above the average (in fact, if we consider the cost associated with not having phone service for one or two days until the line is repaired, this may well be the case for most people in the sample), say that it is \$63.45, then, if $(1 - \pi) = 0.007$, the R index of $[0.993; (0.45, 63)]$ is 2,239. In the areas where the cost of insurance is a bit below 0.45 dollars, say 0.39 dollars and the repair cost is 55.39, the R index of $[0.993; (0.39, 55)]$ is 2,590. These examples show that it is not difficult to find values around the average values of π, g and l that would indicate that many individuals were taking a gamble riskier than Rabin's $[0.5; (105, 100)]$.[4]

An observation regarding the "average gamble" is in order. Consumers were actually facing a more risky gamble than the average one. The reason is that the typical consumer was facing a distribution of repair costs. By replacing this distribution with its average, one is underestimating the riskiness of the true gamble.

2. Evans and Viscusi (1991) use marketing survey data to study the implicit risk-dollar tradeoffs that consumers make when buying certain products. They consider products that may cause some minor health hazard such as insecticides and toilet bowl cleaners. The cheaper insecticides and toilet bowl cleaners are associated with a risk of injury, and the authors find that consumers identify the minor health hazard with a loss in income.

(a) In the case of insecticides, let g be the savings per bottle of a cheaper insecticide

[4] It can also be shown that some individuals in the sample were even taking gambles with negative expected value.

and l the monetary loss associated with the risks of injury (skin poisoning, inhalation and child poisoning). Probabilities are $\pi = 0.9985$ and $(1 - \pi) = 0.0015$ for no-injury and injury per bottle, respectively. Take the gamble $[0.9985; (7.84, 4589)]$ at the average of the data. This gamble has an R index of about 18,000. We thus conclude that the average person in the sample accepts a gamble that is riskier than Rabin's $[0.5; (105, 100)]$.

It is not difficult to consider values around the average values of g, l and the probability π using the standard deviations reported in their article so that we would cover virtually all the individuals in the sample. We would then find even greater differences in riskiness with respect to Rabin's gambles.

(b) In the case of toilet bowl cleaners, let g denote savings per bottle of a cheaper toilet bowl cleaner and l the monetary loss associated with the risks of injury (eye burns, gassings and child poisoning). Probabilities are $\pi = 0.9985$ for no-injury and $(1 - \pi) = 0.0015$ for injury, per bottle. Take the gamble at the average of the data for g and l: $[0.9985; (2.23, 2000)]$. This gamble for the "average consumer" has a negative expected value. This consumer would be taking an unfair gamble. One can easily generate other gambles with negative mean if one plays around with different values of π, g and l that fit well within the sample. It is also easy to find gambles with positive mean that are riskier than Rabin's. For instance, the gamble $[0.999; (2.23, 2000)]$ has an R index of about 9,200.

The next example shows that in an experimental situation using real money, subjects may often accept riskier gambles than the type Rabin (2000) assumes they reject.

3. Henrich and McElreath (2002) report experiments with real money with university undergraduates at UCLA choosing between \$15 for sure and a 50-50 gamble of win \$30 or \$0. More than 80 percent of students chose the risky option, which is effectively to take the gamble $[0.5; (15, 15)]$. When asked to choose between \$15 for sure and a 20-80 gamble of win \$75 or \$0, i.e., the gamble $[0.2; (60, 15)]$, over 52 percent of the students chose the risky gamble. Thus, at least 80 and 52 percent of the students, respectively, were accepting small gambles of zero expected value.[5]

[5]The authors are interested in studying the extent to which small-scale rural farmers in different ethnographic contexts are risk averse or risk lovers. In addition to the UCLA undergraduates, they study the behavior of Mapuche, Huinca, and Sangu peasants in an experimental setting using real money, broadly equivalent to one-third of a day's wage. Most of these peasants, especially the Mapuche, were

We close this section by noting the only empirical evidence that Rabin and Thaler (2001) cite in their favor. They refer to Fehr and Gotte (2002) who report that about 50% of a sample of bicycle messengers in Zurich rejected the gamble $[0.5;(8,5)]$. The R index for this gamble is 13.57. Clearly, all our previous examples exhibit much higher values of the R index. But if the threshold is to show that individuals may accept real gambles with values of the R index greater than just 13.57, then it is easy to find further evidence. For instance, in Holt and Laury (2002) individuals have to choose between two risky options involving different probabilities and stakes. In one of their many treatments individuals were asked to choose between gain 100 with probability 0.7 and gain 80 with probability 0.3 (option A), and gain 192.5 with probability 0.7 and gain 5 with probability 0.3 (option B). Over 40% of individuals chose option B over option A, that is effectively take the gamble $[0.7;(92.5,75)]$, whose R index is 77.77.

In Table I we report the values of the different indices of riskiness discussed in the previous subsection for the several gambles considered here. It is apparent that one reaches similar conclusions using any of these measures.

[Table I about here]

The examples in this section critically question the empirical validity of Rabin's assumption of rejecting small gambles (assumption q) without specifying the decision maker's preferences. We next argue that, while q can be challenged by studying how individuals behave and say they would behave when facing small gambles, it is unclear where one should draw the line to be satisfied with evidence supporting or invalidating Rabin's assumption q. The reason is that in the typical sample there may be both acceptors and rejectors of the same small gamble under consideration. We thus propose to bring to bear empirical evidence obtained under the expected utility assumption. Much of the empirical literature attempts to find a model of preferences, along with other constructs of the models, that be a "good fit" for *all* the observed behavior. As explained for example in Segal and Spivak (1990), any expected utility agent with a differentiable utility function must accept infinitesimal gambles of positive expected value, because locally these preferences

more risk-prone than the UCLA undergraduates.

amount to risk neutrality. However, as soon as the gambles are of any non-infinitesimal size, expected utility is compatible with both accepting and rejecting small gambles. It is then necessary to perform careful econometric analyses to find the best possible fit for the observed behavior.

3 Risk Attitudes under Expected Utility

This section shows that Rabin's assumption q (rejecting small gambles) under p (expected utility) implies a specific positive lower bound on the coefficient of absolute risk aversion. It then shows that in the case of Rabin's gambles, this lower bound turns out to imply that the assumed values of the relative risk aversion coefficient are much higher than the estimates ever obtained in the empirical literature. Thus, the evidence demonstrates that the joint hypothesis "p and q" is empirically invalid. As such, it is not at all surprising that Rabin finds highly paradoxical rejections of large-stakes gambles, that is that "p and q" implies unrealistic behavior. However, our reason to translate Rabin's (2000) joint assumption into the language of risk aversion coefficients is to apply it to his very same exercises. Indeed, the main point of this section is to replicate them, but using in them a joint assumption "p and q" that implies empirically plausible levels of relative risk aversion. In sharp contrast with his result, our finding is that Expected Utility implies then no paradoxical behavior.

3.1 Rejecting Small Gambles: Testable Implication on Risk Aversion Coefficients

For a decision maker with wealth level w and twice continuously differentiable Bernoulli utility function u, denote the Arrow-Pratt coefficient of absolute risk aversion by $\rho_A(w,u) = -\frac{u''(w)}{u'(w)}$, with the coefficient of relative risk aversion being defined as $\rho_R(w,u) = w \cdot \rho_A(w,u)$. Note that $\rho_A(w,u) = \rho_R(w,u) = 0$ for a risk neutral individual, while they are positive for a strictly concave u.

Rabin (2000) shows that if an individual is a risk averse expected utility maximizer [assumption p] and rejects a given gamble of equally likely gain g and loss l, $g > l$, over a given range of wealth levels [assumption q], then he will reject correspondingly larger gambles of gain G and loss L. We show now that the joint assumption "p and q" implies a positive lower bound on $\rho_A(w,u)$. In fact, this positive lower bound can be calculated exactly and, therefore, provides a testable implication of the assumption.

Proposition. Let u satisfy non-increasing absolute risk aversion. Let I be an interval in the positive real line. If for every $w \in I$,

$$\frac{1}{2}u(w+g) + \frac{1}{2}u(w-l) < u(w),$$

there exists $a^* > 0$ such that the absolute risk aversion coefficient $\rho_A(w,u)$ is at least a^* for all $w \in I$. Moreover, the highest such a^* is the solution to the equation

$$f(a) = e^{al} + e^{-ag} - 2 = 0.$$

Proof: Suppose not. Then, for every $a > 0$ there exists $w \in I$ such that $\rho_A(w,u) < a$. In particular, this holds for the unique $a^* > 0$ solving the equation $f(a) = 0$.

Consider the constant absolute risk aversion utility function $v(w) = -e^{-aw}$ for $a < a^*$. For such a choice of a, $f(a) < 0$, i.e.,

$$e^{al} + e^{-ag} < 2,$$

or

$$\frac{1}{2}\left(-e^{-a(w-l)}\right) + \frac{1}{2}\left(-e^{-a(w+g)}\right) > -e^{-aw}.$$

Thus, an individual with utility function v would agree to play the small-stakes lottery with gain g and loss l starting from any wealth level w.

Denote by $w' \in I$ the wealth level for which $\rho_A(w',u) = a$. By non-increasing absolute risk aversion, for $w \in I$, $w \geq w'$, the individual with utility function v is at least as risk averse as the one with utility function u. Therefore, using the well-known characterization of comparisons of risk attitudes across individuals (e.g., Mas-Colell et al. (1995, Chapter 6)), it follows that

$$\frac{1}{2}u\left(w+g\right) + \frac{1}{2}u\left(w-l\right) > u(w)$$

for every $w \in I$, $w \geq w'$, which is a contradiction. ■

The hypothesis of non-increasing absolute risk aversion is generally accepted in the literature. Note, however, that it is not essential to the argument in the above proof. We use it only in the last step to assert that the range of wealth levels over which the "rejecting the small-stakes lottery" assumption is violated constitutes an interval of the form $[w', \infty)$, something stronger than we need. In the absence of the non-increasing absolute risk aversion assumption, continuity of the utility function would suffice to obtain the same result over some arbitrary interval. This would also be enough for our purposes.

Hence, contrary to several statements in Rabin (2000, 2001) and Rabin and Thaler (2001), the conclusion to be drawn from this proposition is that the assumption of rejecting the small-stakes gamble does go beyond concavity of the Bernoulli utility function. A positive lower bound on $\rho_A(w, u)$ is also assumed, and this bound is independent of the interval I over which the assumption is made. Translated to $\rho_R(w, u)$, the bound clearly increases when for a given small-stakes gamble we enlarge the interval I over which it should be rejected. This means for example that the assumption that a given gamble is rejected for all wealth levels is incompatible with the agent becoming risk neutral at some sufficiently high level of wealth, feature shared by a large class of concave utility functions. The proposition implies that $\rho_R(w, u)$ must go to infinity as wealth goes to infinity.[6]

3.2 Empirical Evidence

There is much work, both experimental and empirical, concerning the behavior of expected utility agents towards risks of different magnitudes. We survey this literature in Palacios-Huerta et al. (2001). It will suffice for our purposes here to summarize its main findings: (1) expected utility is many times a good fit to real behavior, even when considering more general theories as alternative hypotheses; and (2) the estimates of $\rho_R(w, u)$ are consistently in the single-digit range.[7]

[6]This is contrary to what is claimed in the literature. For instance, "the calibration theorem is entirely nonparametric, assuming *nothing* about the utility function except concavity" (Rabin, 2000, p. 1282).

[7]The evidence yielding these results includes several studies concerning small, medium and large gambles. This contradicts assertions like "when measuring risk attitudes maintaining the expected-utility hypothesis ... data sets dominated by modest-risk investment opportunities are likely to yield much higher estimates of risk aversion than data sets dominated by larger investment opportunities" (Rabin, 2000, p. 1287).

The pattern on the single-digit estimates of $\rho_R(w, u)$ is remarkable. It has been obtained in many different settings, under very different circumstances and for very different sizes of risk. In sharp contrast with this, Rabin's assumption that an expected utility person turns down gambles where she loses \$100 or gains \$105 for any initial wealth level implies that $\rho_R(w, u)$ must go to infinity when wealth goes to infinity, while the assumption that a 50-50 lose \$100 or gain \$105 bets is turned down for any lifetime wealth level less than \$350,000 implies a value of the same coefficient no less than 166.6 at \$350,000. The strikingly large discrepancy between the size of these coefficients and the ones observed empirically may explain the paradoxes in Rabin (2000).[8]

We shall now tackle the key issue of this section: we pose the question of whether or not rejecting Rabin's gambles over smaller wealth intervals, those corresponding to single-digit values of $\rho_R(w, u)$, induces paradoxical behavior. We study it in the next subsection, and remark that the answer is a priori far from obvious.

3.3 Rabin's Gambles with Single-Digit Values of ρ_R

Although the point we are raising is general, only for computational simplicity we shall work with the class of CRRA (constant relative risk aversion) Bernoulli utility functions $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ for $\gamma \geq 0$. For this utility function, $\rho_R(w, u) = \gamma$.

The assumption of rejecting small-stakes gambles is generally made over a given range of wealth levels. In Table II we calculate, for two small-stakes lotteries and for different values of γ, the largest wealth level at which an individual rejects the lotteries.

[Table II about here]

Note that the values of these wealth levels are extremely small. Therefore, assuming the rejection of these small gambles "from any wealth level" or "for any wealth less than or equal to 350,000" for decision makers with γ in the single digits is empirically irrelevant.

[8] A final observation concerning the evidence is in order. While the theory usually focuses on life-time wealth, empirical studies typically use per period data (e.g., monthly income, yearly consumption, and so on). Under the standard assumption of time stationarity of preferences, one can easily show that estimates of the preference parameters for the per period utility function correspond to those of the utility function over life-time wealth.

Put it differently, individuals' behavior is not consistent with the idea that they are viewing these choice situations in terms of life-time wealth levels.[9]

Continuing with the specification of CRRA utility, the next question we examine is how high is the bound a^* associated with the given small-stakes lottery of gain g and loss l. On the basis of the same lotteries used in Rabin's (2000) Tables II and III, we calculate in Table III their corresponding values of a^*, as defined in the proposition above. The table also shows, for wealth levels of $300,000 and $30,000, the induced values of γ.

[Table III about here]

It is first worth noting that for the wealth level of $300,000 very few values of γ are in the single-digit range, or even in the teens. No single-digit value arises when the gamble involves losing $100 or $1,000. Only when the rejected gamble involves losing $l = \$10,000$, which would not appear to be a small-stakes gamble, such low values start to arise consistently. In an attempt to generate more γ coefficients in the single-digit range, we examine the same lotteries for a wealth level as low as $30,000. In this case, single-digit coefficients arise for some gambles where $l = \$1,000$, and for all gambles where $l = \$10,000$, which are hardly small-stake gambles for an individual with that wealth level. For the lowest stakes gambles involving $l = \$100$ a single-digit γ is only found when $g = 101$. We conclude from Table III that empirically plausible, single-digit, values of γ are compatible with the assumption only when the loss l in the gamble is a significant proportion of the individual's wealth. In other words, the empirically plausible version of assumption q imposes the rejection of the small gambles over a much smaller interval of wealth levels than those used in Rabin (2000).

Next we try to replicate Rabin's results, but imposing that γ be in the single-digit range, which would correspond to imposing assumption q over a suitably smaller interval of wealth levels. We wish to find out whether this still induces paradoxical behavior.

[9]As a referee noted, subjects may view lab choice situations and "short-term" problems in isolation, rather than in conjunction with other sources of income. Moreover, as Rubinstein (2001) notes, "nothing in the von Neumann-Morgenstern (vNM) axioms dictates use of final wealth levels ... vNM are silent about the definition of prizes ... The definition of prizes as final wealth levels is no less crucial to Rabin's argument than the expected utility assumption."

For various lotteries in Table III that yield values of γ in the single-digit range, Table IV replicates the exercise in Rabin (2000) and displays the best large-stakes lottery with gain G and loss L that the decision maker would reject.

[Table IV about here]

It is apparent that these rejections are no longer paradoxical. For instance, for a wealth of $300,000 the agent turns down gambles involving losses L ranging from 5 to 15 percent of his wealth and gains G that appear reasonable. The same can be said for a wealth of $30,000. In this case, note that relative to wealth these values of L are ten times greater than those in Rabin (2000). Thus, not even for these much larger gambles paradoxical behavior is obtained. Finally, it is worth stressing that gambles with $G = \infty$ are turned down only when potential losses L represent a significantly great proportion of the individual's wealth.

The reasonable behavior described in these large-stakes gambles contrasts with the paradoxes in Rabin (2000) and in other authors in the literature, and may be viewed as a further confirmation of the empirical soundness of single-digit values for $\rho_R(w, u)$.

These results refute assertions such as "paradoxical implications are not restricted to particular contexts or particular utility functions," or "within the expected-utility framework, for *any* concave utility function, even very little risk aversion over modest stakes implies an absurd degree of risk aversion over large stakes" (Rabin (2001, p. 203)). That is, much more than "very little risk aversion over modest stakes" is needed to generate paradoxical behavior. Indeed, as we have shown, this is only obtained when the assumption q is made over too large an interval of wealth levels, which corresponds to a region of the parameter space that is not empirically relevant.

4 Concluding Remarks

Using a problem posed to one of his colleagues as a starting point, Samuelson (1963) argues that, under expected utility, the rejection of a given single gamble for all wealth levels implies the rejection of the compound lottery consisting of the single gamble being repeated an arbitrary number of times. Samuelson's exercise sheds light on the fact that some decision makers may be misapplying the law of large numbers when accepting a compound lottery (the colleague's response was that he would reject the single lottery, but accept its compound version). However, Samuelson was clearly aware of the crucial importance of the assumption of rejecting the single lottery for *all* wealth levels or a large range thereof: "I should warn against undue extrapolation of my theorem. It does not say that one must always refuse a sequence if one refuses a single venture: if, at higher income levels the single tosses become acceptable, and at lower levels the penalty of losses does not become infinite, there might well be a long sequence that it is optimal" (p. 112). Indeed, it may very well be the case that Samuelson's colleague was not fooled by any fallacy of large numbers. He simply violated the assumption of rejecting the given small-stakes lottery for all wealth levels or large range thereof.

The main advantage of expected utility is its simplicity and its usefulness in the analysis of economic problems involving uncertainty. As often argued in the literature, its predictions sometimes conflict with people's behavior. This has led economists to develop various non-expected utility models which can often accommodate actual behavior better. The non-expected utility research agenda is an important one, and there is no question that we should continue to pursue it. However, expected utility should not be accused when it is not at fault. The analysis in this paper shows how certain paradoxical examples in the literature are many times counterfactuals.

In a more recent paper, Rabin and Thaler (2001) continue to drive home the theme of the demise of expected utility and compare expected utility to a dead parrot from a Monty Python show. To the extent that all their arguments are based on introspection and on the calibrations in Rabin (2000), not on empirical evidence, the expected utility parrot may well be saying that "the report of my death was an exaggeration."

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TABLE I

Indices of Riskiness for Different Gambles

Gamble	Probability of Gain π	Gain g	Loss l	Mean M	Variance V	V/M	R index	w index	Yaari's l/α^* index
Rabin (2000)									
Gamble 1	0.5	105	-100	2.5	10,506.3	4,202.5	2,100.0	1,050.6	96.5
Gamble 2	0.5	11	-10	0.5	110.2	220.5	110.0	55.1	9.3
Cicchetti and Dubin (1994)									
Avg. values	0.995	0.45	-54.55	0.175	15.0	85.9	59.6	57.3	0.6
Deviations in π and l	0.993	0.45	-63	0.006	27.9	4,783.5	2,239.0	1,227.3	62.1
Deviation in π and g	0.993	0.39	-55	0.002	21.3	9,394.7	2,590.0	2,376.0	54.6
Evans and Viscusi (1991)									
Insecticide (avg. values)	0.9985	7.84	-4,589	0.945	31,648.9	33,500.0	18,000.0	10,823.3	4,035.5
Toilet bowl cleaner (avg. values)*	0.9985	2.23	-2,000	-0.773	6,004.3				
Toilet bowl cleaner (deviation in π)	0.999	2.23	-2,000	0.228	4,004.9	17,583.0	9,200.0	5,451.6	1,795.6
Henrich and McElreath (2002)									
Gamble 1*	0.5	15	-15	0	225.0				
Gamble 2*	0.2	60	-15	0	900.0				
Fehr and Gotte (2002)	0.5	8	-5	1.5	42.2	28.1	13.5	7.0	3.6
Holt and Laury (2002)	0.7	92.5	-75	42.25	5,891.8	139.4	77.7	89.0	33.2

*Note: We only report values of the indices for gambles with positive mean. For the others, some of the indices are not well defined.

TABLE I I

Wealth levels at which an individual with CRRA (γ) utility function stops rejecting a 50-50 lose \$100 / gain \$*g* lottery.

	g	
γ	125	110
2	400	1,000
3	1,501.3	3,300.5
4	2,003.1	4,401.2
5	2,504.9	5,502
6	3,006.9	6,602.7
7	3,508.8	7,703.5
8	4,010.8	8,804.3
9	4,512.8	9,905.1
10	5,014.9	11,006
11	5,516.9	12,106.8
12	6,018.9	13,207.6
20	10,035.4	22.014.2
30	15,056	33,022.5
40	20,076.7	44,030.8
50	25,097.3	55,039.2

TABLE III

Lower bounds on the coefficient of absolute risk aversion for an individual that rejects a 50-50 lottery lose $*l* / gain $*g* (a*) for any range of wealth levels, and associated lower bound on the coefficient of relative risk aversion for wealth levels $300,000 and $30,000.

l / *g*	a*	γ = 300,000a*	γ = 30,000a*
100 / 101	.0000990	29.7	2.9
100 / 105	.0004760	142.8	14.2
100 / 110	.0009084	272.5	27.2
100 / 125	.0019917	597.5	59.7
100 / 150	.0032886	986.5	98.6
1,000 / 1,050	.0000476	14.2	1.4
1,000 / 1,100	.0000908	27.2	2.7
1,000 / 1,200	.0001662	49.8	4.9
1,000 / 1,500	.0003288	98.6	9.8
1,000 / 2,000	.0004812	144.3	14.4
10,000 / 11,000	.0000090	2.7	0.2
10,000 / 12,000	.0000166	4.9	0.4
10,000 / 15,000	.0000328	9.8	0.9
10,000 / 20,000	.0000481	14.4	1.4

TABLE IV

If averse to 50-50 lose \$*l* / gain \$*g* for wealth levels \$300,000 and \$30,000 with CRRA utility and coefficient of relative risk aversion γ, will also turn down a 50-50 lose *L* / gain *G* bet; *G*'s entered in Table.

Wealth: \$30,000

l/g:	100/101	1,000/1,050	1,000/1,100
γ:	2.9	1.4	2.7
L			
400	416	---	---
600	636	---	---
800	867	---	---
1,000	1,107	---	---
2,000	2,479	2,205	2,439
4,000	6,538	4,917	6,259
6,000	14,538	8,329	13,168
8,000	40,489	12,749	30,239
10,000	∞	18,686	495,319

Wealth: \$300,000

l/g:	10,000/11,000	10,000/12,000
γ:	2.7	4.9
L		
15,000	17,341	19,887
17,000	20,072	23,572
20,000	24,393	29,792
22,000	27,435	34,490
25,000	32,266	42,574
30,000	41,116	59,870
40,000	62,594	126,890
50,000	91,268	∞
75,000	239,089	∞