On a Controversy Regarding Expected Utility and Risk Aversion: Individuals Accept Small Gambles*

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Abstract

This paper contributes to a recent debate regarding expected utility and risk aversion. Based on introspective rejections of small gambles, expected utility has been deemed incapable of explaining plausible behavior toward risk. We use empirical and experimental evidence to show that people often accept small gambles that are riskier than those used in the introspective rejections. A first step of our analysis is the development of a measure of riskiness of a gamble.

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1 Introduction

Expected utility (EU) is the leading paradigm in the theory of decision making under uncertainty and in game theory. In it, risk aversion is identified with concavity of the utility function, which implies that the utility change after a gain is smaller than after a loss of equal value. The EU framework has been severely criticized in a recent literature that concludes that concavity of the utility function is not a good representation of risk aversion.[1] The basis of the criticism can be best illustrated in Rabin (2000), who studies the relationship between risk attitudes over small and large stakes gambles under EU. Using his results, it is possible to present striking statements of the following kind: if a decision maker is a risk-averse EU maximizer and if he rejects gambles involving small stakes over a large range of wealth levels, then he will also reject gambles involving large stakes, sometimes with infinite positive returns. For instance, "suppose that, from any initial wealth level, a person turns down gambles where she loses $100 or gains $110, each with 50% probability. Then she will turn down 50-50 bets of losing $1,000 or gaining *any* sum of money," or "suppose we knew a risk averse person turns down 50-50 lose $100 or gain $105 bets for any lifetime wealth level less than $350,000 ... Then we know that from an initial wealth level of $340,000 the person will turn down a 50-50 bet of losing $4,000 and gaining $635,670." (Rabin, 2000, p. 1282).

From this paradoxical, even absurd, behavior towards large-stakes gambles, Rabin (2000) and other authors conclude that EU is fundamentally unfit to explain decisions under uncertainty. This paper challenges this conclusion. We show that the paradoxes identified in Rabin (2000) have little empirical support.[2] In particular, we show that it is the assumption of rejecting small gambles over a large range of wealth levels, and *not* EU, that does not typically match real-world behavior.

It is surprising that the plausibility of "rejecting small gambles" as an assumption is argued in the literature purely by appealing to the reader's introspection. Introspection,

[1]See, for example, Hansson (1988), Kandel and Stambaugh (1991), Rabin (2001), Rabin and Thaler (2001) and other references therein. Samuelson (1963), Machina (1982), Segal and Spivak (1990) and Epstein (1992) also study various issues that are related to this literature.

[2]See Cox and Sadiraj (2001), Rubinstein (2001), and Watt (2002), who have provided arguments related to ours. See also our companion paper, Palacios-Huerta et al. (2001).

however, may sometimes be misleading: what people think they *would do* in a thought experiment may turn out to be quite different from what they *actually do* when confronted with a similar situation in which real payments are involved. Indeed, we shall see that the assumption is far from being empirically evident.

Rather than relying on introspection, we use empirical and experimental evidence from small gambles that individuals face, to show how in practice people often accept gambles that are *as risky* or even *riskier* than the ones Rabin (2000) says they reject. That is, the "rejecting small gambles" assumption can be directly challenged on empirical grounds. As a first step in our analysis, to be able to talk about gambles that are riskier than those in Rabin (2000), we develop next a measure of riskiness of a gamble.

2 The Riskiness of a Gamble

The most uncontroversial notion of riskiness is given by the concept of second-order stochastic dominance (SOSD); see, e.g., Rothschild and Stiglitz (1970) and Mas-Colell et al. (1995, Chapter 6). However, this incomplete ordering would render most of the gambles we will consider non-comparable and, therefore, it is not very useful. It turns out, though, that in the examples of Section 3 we can make our point using several more crude and intuitive measures of riskiness, which will allow to order more gambles. We begin with some basic definitions.

A *gamble* can be identified with a cumulative distribution function $F(x)$ over the prizes x, where $x \in \mathbf{R}$. Thus, for any x, $F(x)$ expresses the probability of receiving a prize less than or equal to x. For simplicity, we concentrate on gambles with finite support $\{x_1, \ldots, x_n\}$, $x_i < x_{i+1}$ for $i = 1, \ldots, n-1$. The probability of prize x_i is π_i for $i = 1, \ldots, n$. We write a gamble as $[\pi; x]$, where $\pi = (\pi_1, \ldots, \pi_n)$ and $x = (x_1, \ldots, x_n)$. We shall also restrict attention to gambles of positive mean: measuring the riskiness of a gamble is most interesting when at least it offers positive expected returns to compensate for risk.

The Bernoulli *utility function* u assigns to each level of the decision maker's final wealth w a real number. We shall assume that it is twice continuously differentiable. For the utility function u, the Arrow-Pratt (Arrow (1971), Pratt (1964)) *coefficient of absolute* *risk aversion* is $\rho_A(w,u) = -\frac{u''(w)}{u'(w)}$. This measures the decision maker's local attitude towards risk for each wealth level w; see, for example, Mas-Colell et al. (1995, Chapter 6).

A specific kind of preferences will play an important role in our analysis; it is the *constant absolute risk aversion* (CARA) family, whose Bernoulli utility function is $u(w) = -\exp -aw$, where $a > 0$ is a constant indicating this decision maker's absolute risk aversion at every level of wealth.

Given a gamble $[\pi; x]$ with positive expected value, we shall define its *riskiness* as the inverse of the absolute risk aversion coefficient for the CARA individual that is indifferent between accepting and rejecting the gamble. That is,

$$R[\pi; x] = \frac{1}{a^*},$$

where $a^* > 0$ solves $h(a^*) = \sum_{i=1}^{n} \pi_i \exp -a^* x_i - 1 = 0$.

Note that a^* is well defined since this equation has a unique positive solution. Indeed, if all prizes in $[\pi; x]$ are non-negative, $a^* = \infty$ (and consequently, $R[\pi; x] = 0$ in this case). Otherwise, to see that $a^* > 0$ is well defined, it suffices to use the intermediate value theorem after observing that h is continuous, $h(0) = 0$, $h(\infty) = \infty$, and $h'(a)$ is strictly increasing and takes a negative value at $a = 0$: $h'(0) = -\sum_{i=1}^{n} \pi_i x_i < 0$.

In the absence of an axiomatic derivation of a measure of riskiness, we will enumerate some desirable intuitive properties that this proposed measure $R[\pi; x]$ possesses:

- (i) Consider a gamble and denote its riskiness by R. Then, the set of EU maximizers whose coefficient of absolute risk aversion $\rho_A(w,u) < 1/R$ for all w will accept the gamble, while the set of EU maximizers whose $\rho_A(w,u) > 1/R$ for all w will reject it.[3]

- (ii) Homogeneity of degree 1 in prizes: for every $\lambda > 0$, $R[\pi; \lambda x] = \lambda R[\pi; x]$.

Since in the current paper we are concerned with the claims in Rabin (2000), we shall restrict attention to binary gambles, as he also does. For binary gambles, π_1 is the

[3]This property follows from the definition of $R[\pi; x]$ and the characterization of risk aversion across individuals (e.g., Mas-Colell et al. (1995, Proposition 6.C.2)).

probability of the loss x_1, while $\pi_2 = 1 - \pi_1$ is the probability of the gain x_2. Therefore, let us denote a binary gamble as $[\pi_1; (x_1, x_2)]$. Then, one can list some additional properties of the index R:

- (iii) Strictly decreasing in the probability of gain: if $\pi_1 < \pi_1'$, $R[\pi_1; (x_1, x_2)] > R[\pi_1'; (x_1, x_2)]$.
- (iv) Strictly decreasing in the gain: if $x_2 > x_2'$, $R[\pi_1; (x_1, x_2)] < R[\pi_1; (x_1, x_2')]$.
- (v) Strictly increasing in the loss: if $x_1' > x_1$, $R[\pi_1; (x_1', x_2)] < R[\pi_1; (x_1, x_2)]$.

Therefore, we shall say that a gamble with positive expected value is more risky than another if it has a higher value of the index R. For our purposes, this is a simple statistic that can be computed on all the gambles we will study. Its dimension is the one of the random variable, implying for example that Rabin's $[0.5; (-100, 105)]$ gamble is 10 times more risky than the $[0.5; (-10, 10.50)]$ gamble, a property that seems intuitive for the current discussion. In addition, this index gives us a clean measure of riskiness that abstracts from wealth considerations.[4]

3 Accepting Small Gambles: Evidence

Next we proceed to calculate the values of R for different small gambles. For us, as for Rabin (2000) and the related literature, "small gambles" simply means that the stakes are a small fraction of consumers' wealth, or also that their expected value, while positive, is small.

First, we report the values of R for different gambles used in the introspective arguments in Rabin (2000) and Rabin and Thaler (2001). Consider the gamble $[0.5; (-10, 11)]$. It has a value of R of 110.08. The gamble $[0.5; (-100, 105)]$ has a value of R of 2,100.

[4]The R index just defined is not the only measure of riskiness we could use in the examples of Section 3 to show that many people accept gambles that are riskier than Rabin's. For instance, one could also define riskiness on the basis of other one-parameter preferences (such as the constant relative risk aversion family, or the Yaari (1987) dual theory family), or appeal to related notions found in the literature, such as the variance/mean ratio.

Next we report the values of R for gambles taken from some daily life and lab experiences. They show that individuals may well accept gambles whose riskiness, as measured by the R index, is greater than 2,100.

1. Cicchetti and Dubin (1994) investigate the decision of whether to purchase insurance against the risk of telephone line trouble in the home for approximately 10,000 residential customers. The price of this insurance was \$0.45 a month. There is a range for the values of the cost of repair in case of trouble, depending on its seriousness and on the quality and age of the installation. The paper makes no reference to individual data, but it makes it clear that this cost of repair was always below \$100 for all consumers. Conceivably, the decision problem facing a typical user in the area was whether she should pay a fixed \$0.45 extra a month as insurance, or decline it and take the corresponding gamble, whose consequences would be to pay for a repair no more expensive than \$100. The only remaining question is the probability of line trouble, estimated empirically to be about 0.005, ranging from 0.00318 to 0.00742. It is known that a 42.9% of users in the sample chose not to buy insurance, so these users were taking a small gamble. The following gambles correspond to values of the probability and the cost of repair that fall well within the sample of users not buying insurance. The values of the R index are also reported:

- $[0.007; (-63, 0.45)]$, with $R = 2,412$;
- $[0.006; (-74, 0.45)]$, with $R = 5,033$;
- $[0.005; (-89, 0.45)]$, with $R = 7,267$.

The reader can check that the mean of these gambles is positive. This shows that one can find many data points in the sample that correspond to users taking gambles that are riskier than those used by Rabin (2000) and Rabin and Thaler (2001). There are also other points in the sample that yield lower values of R, for example, $[0.005; (-55, 0.45)]$ yields $R = 62$. Accepting a gamble like this does not constitute an argument against Rabin's claims, because its riskiness is lower. In terms of the size of the loss, these gambles are roughly comparable to those proposed by Rabin (2000). There is one aspect, though, that makes them significantly different, and that is the fact that the probability distribution in the gamble is far from the uniform assumed by Rabin. Our next paragraph will address this point.

2. Gertner (1993) studies the betting behavior of contestants in the T.V. game show "Card Sharks." The paper focuses on the final round of betting. Before that round, the typical contestant has accumulated an amount of money, say M dollars. A card is drawn from a deck and shown publicly; a second card will be drawn from the full deck after the contestant wagers some amount of mony. The contestant is asked to wager some amount no smaller than $M/2$. Call his wager x. The gamble he faces has three possible outcomes: if the value of the new card exceeds that of the first, the contestant will gain x; if it is lower, he will lose x, while he will gain or lose nothing if the value of the second card is equal to the first. For example, if the first card drawn is a '6,' the probability of gain is $8/13$ (that of drawing cards of values $7, 8, 9, 10, J, Q, K, A$), the probability of loss is $4/13$ (that of drawing cards of values $2, 3, 4, 5$), while the probability of no gain or loss is always $1/13$. Note how these are objective probabilities, and their values are closer to the values used by Rabin (2000). Denote by $L(v)$ the gamble facing the contestant when the first card drawn has value v. For each $L(v)$, we provide a wager near the average wager across the contestants in the sample, which is fairly close to the actual wager chosen by many of them. Thus, for each $L(v)$ where the probabilities are not extreme, we write down the gamble and the corresponding value of the index R:

- $L(7) : [(5/13, 1/13, 7/13); (-2000, 0, 2000)]$ with $R = 5,944.03$;
- $L(6) : [(4/13, 1/13, 8/13); (-1700, 0, 1700)]$ with $R = 2,452.58$;
- $L(5) : [(3/13, 1/13, 9/13); (-2000, 0, 2000)]$ with $R = 1,820.48$;
- $L(4) : [(2/13, 1/13, 10/13); (-2450, 0, 2450)]$ with $R = 1,522.27$.

As measured by the index R, all these gambles give values comparable to Rabing's $[0.5; (-100, 105)]$. Both $L(6)$ and $L(7)$ are actually riskier. All of them are far riskier than $[0.5; (-10, 11)]$, used in Rabin and Thaler (2001) to kill the EU parrot.[5] Take $L(6)$.

[5]We refer the reader to Rabin and Thaler (2001) and Palacios-Huerta et al (2001) to learn more about the death and resurrection of the parrot. Rubinstein (2001) also provides an appraisal of this parrot

It is reported that 47% of contestants wagered more than the minimum allowed of $M/2$. Making a revealed preference argument under preferences that are not far from EU, if an agent preferred to wager $x > M/2$ to simply wagering the minimum $M/2$, he would also have preferred x to wagering 0. Thus, while we know that those wagering more than the minimum would have accepted the gamble, it is still possible that among the set of contestants that wagered the minimum, a number of them would also prefer this kind of gamble to no gamble at all. The percentage of contestants that wagers the minimum allowed is increasing with v, as one would expect, but the proportion of them betting over the minimum is never below 25%.

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We close the paper by noting the only empirical evidence that Rabin and Thaler (2001) cite in their favor.[6] They refer to Fehr and Gotte (2002) who report that about 50% of a sample of bicycle messengers in Zurich rejected the gamble $[0.5; (-5, 8)]$. The R index for this gamble is 13.57. Clearly, all our previous examples exhibit much higher values of the R index. But if the threshold is to show that individuals may accept real gambles with values of the R index greater than just 13.57, then one can use further evidence.

3. Holt and Laury (2002) study the choice of individuals between two risky options involving different probabilities and stakes. The exercise performed is to offer individuals two gambles, one more risky than the other in terms of the dispersion of prizes. For several values of π, individuals were asked to choose either gain 180 with probability π and gain 144 with probability $1 - \pi$ (option A, the safer option), or gain 346.5 with probability π and gain 9 with probability $1 - \pi$ (option B, the riskier option). Note that if an individual chooses option B over A, he is turning down a gamble that offers him $144 in the worst case. We shall calculate the R index of the transformed option B gamble, where we subtract the small prize in option A from each prize in option B (recall that our calculations of the R index of a gamble always compare that gamble to no gamble at all). In the example above, our transformed option B would be to gain $346.5 - 144 = 202.5$ with probability π and $9 - 144 = -135$ with probability $1 - \pi$), while the transformed option A would be

debate. We will not repeat those passages here; that's what parrots do best. On the other hand, Rabin and Thaler (2001) are a good repetition of the arguments in Rabin (2000).

[6] Rabin (2000) does not cite any empirical evidence.

to gain $180 - 144 = 36$ with probability π and gain 0 with probability $1 - \pi$. A CARA agent would exhibit the same behavior in the choice between A and B and in the choice between their transformed counterparts. Such an individual choosing the transformed B over the transformed A would certainly choose the transformed gamble B over no gamble at all. It is in this sense that our calculations of the R index for the transformed option B are underestimated. That is, the turned down alternative for someone choosing the transformed option B clearly dominates the status-quo of no gamble at all.

We report four different transformed B gambles and their corresponding values of R:

- $L_1 = [0.4; (-135, 202.5)]$ with $R = 202.379$;
- $L_2 = [0.3; (-135, 202.5)]$ with $R = 128.995$;
- $L_3 = [0.2; (-135, 202.5)]$ with $R = 88.5424$;
- $L_4 = [0.1; (-135, 202.5)]$ with $R = 59.4098$.

It is reported that around 20% of subjects chose the option B over the option A that would correspond to L_1, 35% for the one corresponding to L_2, 55% for L_3 and 60% for L_4. For CARA preferences, these percentages of subjects who chose option B over A are also an underestimate of the ones that would choose the respective transformed option B over no gamble at all.

All these examples critically question the empirical validity of Rabin's assumption of rejecting small gambles, at least for significant groups of individuals. For such groups, EU cannot be discarded as a theory that would explain and predict their behavior. When one takes into account the entire sample of agents, one will typically find acceptors and rejectors of the gambles in question. Then, More stringent tests are called for, based on econometric methodology; we refer the reader to our companion paper for a discussion of such issues.[7]

[7] Quite apart from the current debate, which concerns gambles of positive mean, Friedman and Savage (1948) point out the pervasive presence in society of gambling and other unfair lotteries; see also Evans and Viscusi (1991) and Henrich and McElreath (2002) for more evidence concerning people taking gambles of non-positive mean.

REFERENCES

Arrow, K. J. *Essays in the Theory of Risk Bearing.* Chicago, IL: Markham Publishing Company, 1971.

Cicchetti, C. J. and J. A. Dubin. "A Microeconometric Analysis of Risk Aversion and the Decision to Self-Insure." *Journal of Political Economy* 102 (1994): 169-186.

Cox, J. C. and V. Sadiraj, "Risk Aversion and Expected-Utility Theory: Coherence for Small- and Large-Stakes Gambles." Working Paper 01-03, Department of Economics, University of Arizona, 2001.

Epstein, L. G. "Behavior under Risk: Recent Developments in Theory and Applications." In J. J. Laffont (ed.) *Advances in Economic Theory. VI World Congress of the Econometric Society (vol. II)*, Cambridge University Press, 1992.

Evans, W. N. and W. K. Viscusi. "Estimation of State-Dependent Utility Functions Using Survey Data." *Review of Economics and Statistics* 73 (1991): 94-104.

Fehr, E. and L. Gotte. "Do Workers Work More if Wages Are High? Evidence from a Randomized Field Experiment," Working Paper No. 125, Institute for Empirical Economic Research, University of Zurich, 2002.

Friedman, M. and L. J. Savage. "The Utility Analysis of Choices Involving Risk." *Journal of Political Economy* 56 (1948): 270-304.

Gertner, R. "Game Shows and Economic Behavior: Risk Taking on "Card Sharks"." *Quarterly Journal of Economics* 108 (1993), 507-521.

Hansson, B. "Risk Aversion as a Problem of Cojoint Measurement." In *Decision, Probability, and Utility*, ed. by P. Gardenfors and N.-E. Sahlin. Cambridge: Cambridge University Press, 1988.

Henrich, J. and R. McElreath. "Are Peasants Risk-Averse Decision Makers?" *Current Anthropology* 43 (2002): 172-181.

Holt, C. A. and S. K. Laury. "Risk Aversion and Incentive Effects." *American Economic Review* 92 (2002): 1644-1655.

Kandel, S., and R. F. Stambaugh. "Asset Returns, Investment Horizon, and Intertemporal Preferences." *Journal of Monetary Economics* 27 (1991): 39-71.

Machina, M. "Expected Utility Analysis without the Independence Axiom." *Econometrica* 50 (1982): 277-323.

Mas-Colell, A., M. D. Whinston and J. R. Green. *Microeconomic Theory*, Oxford University Press, 1995.

Palacios-Huerta, I., R. Serrano and O. Volij, "Rejecting Small Gambles under Expected Utility," Working Paper 2001-05, Department of Economics, Brown University and Economics Working Paper 32, Institute for Advanced Study, http://www.sss.ias.edu/papers/econpaper32.pdf, 2001.

Pratt, J. "Risk Aversion in the Small and in the Large." *Econometrica* 32 (1964): 122-136.

Rabin, M. "Risk Aversion and Expected-Utility: A Calibration Theorem." *Econometrica* 68 (2000): 1281-1292.

Rabin, M. "Diminishing Marginal Utility of Wealth Cannot Explain Risk Aversion." In *Choices, Values, and Frames.* D. Kahneman and A. Tversky, eds. Cambridge University Press, 2001.

Rabin, M. and R. H. Thaler. "Anomalies. Risk Aversion." *Journal of Economic Perspectives* 15 (2001): 219-232.

Rothschild, M. and J. E. Stiglitz. "Increasing Risk: I. A Definition." *Journal of Economic Theory* 2 (1970): 225-243.

Rubinstein, A. "Comments on the Risk and Time Preferences in Economics." Mimeo (2001), Tel Aviv and Princeton University.

Samuelson, P. A. "Risk and Uncertainty: A Fallacy of Large Numbers." *Scientia* 98 (1963): 108-113.

Segal, U. and A. Spivak. "First Order versus Second Order Risk Aversion." *Journal of Economic Theory* 51 (1990): 111-125.

Watt, R. "Defending Expected Utility Theory." *Journal of Economic Perspectives* 16 (2002): 227-229.

Yaari, M. E. "The Dual Theory of Choice under Risk." *Econometrica* 55 (1987): 95-115.